LECTURE NOTES ON

HYDRAULIC MACHINES

5TH SEMESTER
MECHANICAL ENGINEERING

PREPARED BY:-
Mr. A. SUDHIR KUMAR
LECTURER
GOVT. POLYTECHNIC, KORAPUT

Hydraulic Machines — Turbines

Hydraulic Machines: Hydraulic machines are defined as those machine which convert hydraulic energy into mechanical energy.

Turbines: The hydraulic machine which convert the hydraulic energy into mechanical energy are called turbines.

- This mechanical energy is used in running an electric generator which is directly coupled to the shaft of the turbine.
- Hence the mechanical energy is converted into electrical energy.
- The electric power obtained from the hydraulic energy is known as Hydro-electric power.

Definitions of Heads and Efficiencies of a Turbine

1. **Gross Head**: The distance between the head race level and tail race level when no water is flowing is known as Gross Head. It is denoted by 'H_g'.

2. **Net Head**: It is also called effective head and is defined as the head available at the inlet of the turbine. When water is flowing from head race to the turbine, a loss of head due to friction between the water and the penstock occurs. If h_f is head loss due to friction between penstock and water then

$$\text{Net head } (H) = H_g - h_f$$

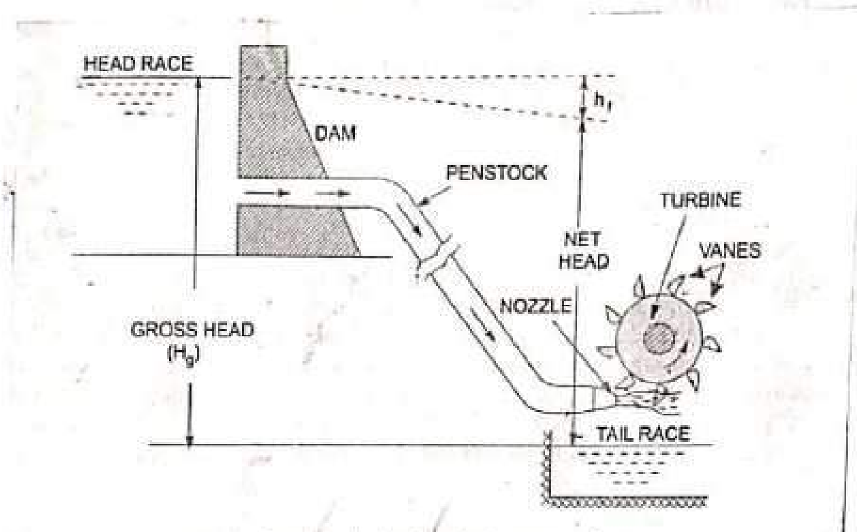
Where H_g = Gross head

$$h_f = \frac{4 \times f \times L \times V^2}{D \times 2g}$$

in which V = Velociy of flow in penstock.

L = Length of penstock.

D = Diameter of Penstock.



Layout of a hydro-eletric power plant.

Classification of Hydraulic Turbines:

1. According to the type of energy at inlet
(a) Impulse turbine (b) Reaction turbine.

2. According to the direction of flow through runner:
(a) Tangential flow turbine (b) Radial flow turbine
(c) Axial flow turbine (d) Mixed flow turbine

3. According to the head at inlet of turbine.
(a) High head turbine (b) Medium head turbine (c) Low head turbine

4. According to the specific speed of the turbine:
(a) Low specific speed turbine (b) Medium specific speed turbine
(c) High specific speed turbine

Impulse Turbine: If at the inlet of the turbine, the energy available is only kinetic energy, the turbine is known as Impulse turbine.

Reaction Turbine: If at the inlet of the turbine, the water possesses kinetic energy as well as pressure energy the turbine is known as reaction turbine.

Tangential flow turbine: If the water flows along the tangent of the runner, the turbine is known as tangential flow turbine.

Radial flow turbine: If the water flows in the radial direction through the runner, the turbine is called radial flow turbine.

Inward flow radial turbine: If the water flows from outward to inward radially, the turbine is known as inward radial flow turbine.

Outward radial flow turbine: If water flows radially from inwards to outwards, the turbine is known as outward radial flow turbine.

Axial flow turbine: If the water flow through the runner along the direction parallel to the axis of rotation of the runner, the turbine is called axial flow turbine.

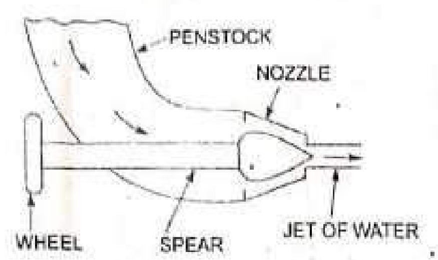
Mixed flow turbine: If the water flows through the runner in the radial direction but leaves in the direction parallel to axis of rotation of the runner, the turbine is called mixed flow turbine.

Pelton Wheel :

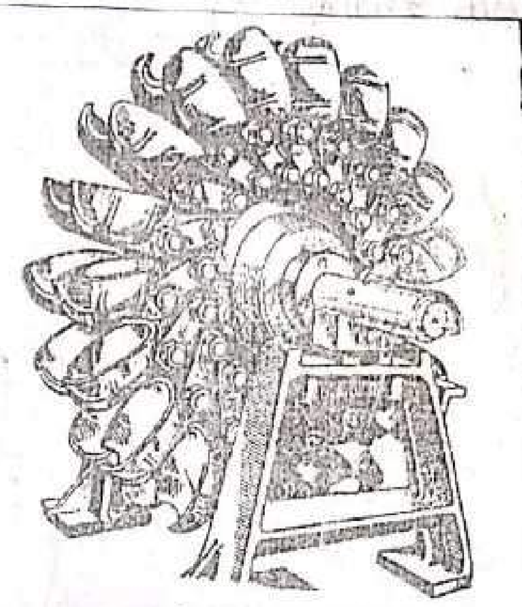
- Pelton Wheel is a tangential flow impulse Turbine as the water strikes the bucket along the tangent of the runner.
- The energy available at the inlet of the turbine is only Kinetic energy.
- It is named as pelton turbine after an American Engineer L. A Pelton.
- This turbine is used for high heads.
- The main parts of the Pelton turbine are as follows:

1. Nozzle and flow Regulating Arrangement: The amount of water striking the buckets (vanes) of the runner is controlled by providing a Spear in the nozzle as shown in fig given below. The spear is a conical needle which is operated either by hands or automatically. When the Spear is pushed forward into the nozzle the amount of water striking the runner is reduced and vice-versa.

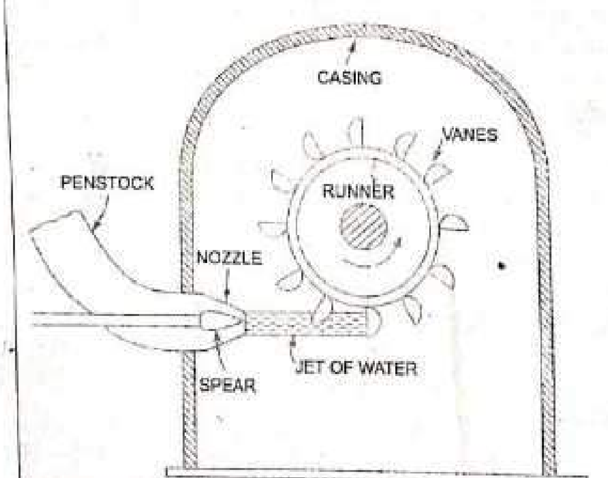
2. Runner with Buckets: It consist of circular disc on the periphery of which a number of buckets evenly spaced are fixed. The shape of the bucket is of a double hemispherical cup which is divided into two symmetrical parts by a dividing wall which is known as splitter. The buckets are shaped in such a way that the jet gets deflected through 160° or 170°. The buckets are generally made of cast iron, cast steel broze or stainless steel depending upon the head at inlet of the turbine.



Nozzle with a spear to regulate flow.


Runner of a pelton wheel.

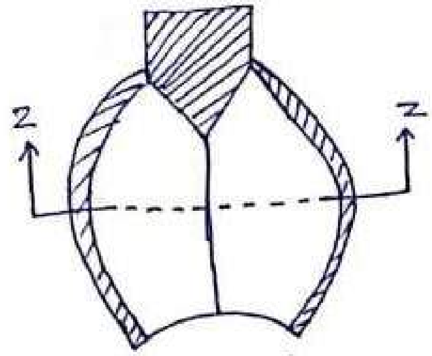
3. Casing : The function of casing is to prevent the splashing of water and to discharge water to tail race. It also acts as Safeguard against accidents. It is made of cast iron or fabricated Steel plates

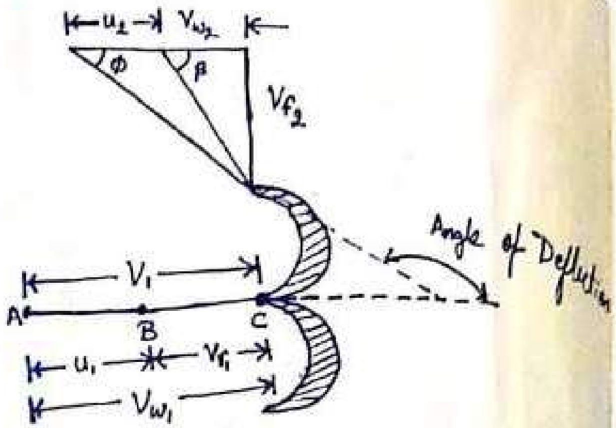


Pelton turbine.

4. Breaking Jet : When the nozzle is completely closed by moving the spear in forward direction, the amount of water striking the runner reduces to zero. But the runner due to inertia goes on revolving for a long time. To stop the runner in a short time, a small nozzle is provided which directs the jet of water on the back of vanes known as breaking jet.

Velocity triangle and Work done for Pelton Wheel





Let H = Net head acting on the Pelton Wheel

$= H_g - h_f$

where H_g = Gross Head and $h_f = \dfrac{4fLV^2}{D_p \times 2g}$

D_p = Diameter of Penstock N = Speed of Wheel in r.p.m

D = Diameter of Wheel d = Diameter of jet of water

V_1 = Velocity of jet at inlet $= \sqrt{2gH}$

$$u = u_1 = u_2 = \frac{\pi D N}{60}$$

The Velocity triangle at inlet will be a straight line

$$V_{r_1} = V_1 - u_1 = V_1 - u$$

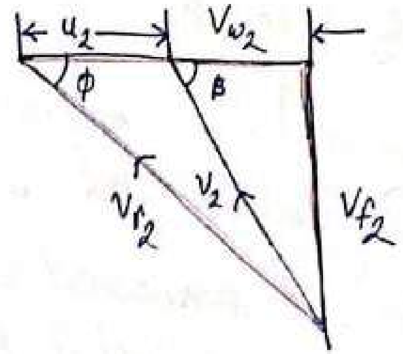
$$V_{w_1} = V_1$$

$$\alpha = 0^\circ \text{ \& } \theta = 0^\circ$$

From velocity triangle at outlet

$$V_{r_1} = V_{r_2}$$

In the outlet triangle



$$\cos\phi = \frac{u_2 + V_{w_2}}{V_{r_2}}$$

$$\Rightarrow \quad V_{r_2}\cos\phi = u_2 + V_{w_2}$$

$$\Rightarrow \quad V_{w_2} = V_{r_2}\cos\phi - u_2$$

Force exerted by the jet of water in the direction of motion $(F_x) = \rho a V_1 [V_{w_1} + V_{w_2}]$

As the angle β is an acute angle, +ive sign is taken.

Also in case of series of vane the mass of water striking is $\rho a V_1$ and not $\rho a V_{r_1}$.

Area of the jet $(a) = \frac{\pi}{4} d^2$

Work done by the jet on the runner per second $= F_x \times u$

$$= \rho a V_1 [V_{w_1} + V_{w_2}] \times u \ \text{Nm/s}$$

Power given to the runner by the jet $= \dfrac{\rho a V_1 [V_{w_1} + V_{w_2}] \times u}{1000}$ kW

Work done/s per unit weight of water striking/s $= \dfrac{\rho a V_1 [V_{w_1} + V_{w_2}] \times u}{\text{Weight of water striking/s}}$

$$= \frac{\rho a V_1 [V_{w_1} + V_{w_2}] u}{\rho a V_1 \times g} = \boxed{\frac{1}{g}(V_{w_1} + V_{w_2}) u} \quad \text{—} (1)$$

Working of Pelton Turbine:

- The water is transferred from the high head source through a long conduit called Penstock.
- Nozzel arrangement at the end of penstock helps the water to accelerate and it flows out as a high speed jet with high Velocity and discharged at atmosphere pressure.
- The jet will hit the splitter of the bucket which will distribute the jet into two halves of bucket and the wheel starts revolving.
- The kinetic energy of the jet is reduced when it hits the bucket and also due to spherical shape of buckets the directed jet will change its direction and takes U-turn and falls into tail race.
- In general the inlet angle of jet is in between 1° to 3°, after hitting the buckets the deflected jet angle is in between 165° to 170°.

Scanned with CamScanner

The energy supplied to the jet at inlet is in the form of Kinetic energy $= \frac{1}{2} m v^2$

$\therefore$ K.E of jet per second $= \frac{1}{2}(\rho a V_1) \times V_1^2$

$\therefore$ Hydraulic efficiency, $\eta_h = \dfrac{\text{Work done per second}}{\text{K.E of jet per second}}$

$$= \frac{\rho a V_1 [V_{w_1} + V_{w_2}]}{\frac{1}{2}(\rho a V_1) \times V_1^2}$$

$$\boxed{\eta_h = \frac{2[V_{w_1} + V_{w_2}] \times u}{V_1^2}} \quad \text{—— (2)}$$

as $V_{w_1} = V_1$, $V_{r_1} = V_1 - u_1 = (V_1 - u)$ + $V_{r_1} = V_{r_2}$

$V_{r_2} = (V_1 - u)$

$V_{w_2} = V_{r_2} \cos\phi - u_2 = V_{r_2} \cos\phi - u = (V_1 - u)\cos\phi - u$

Substituting the values of V_{w_1} and V_{w_2} in the equation (2)

$$\eta_h = \frac{2[V_1 + (V_1 - u)\cos\phi - u] \times u}{V_1^2}$$

$$= \frac{2[(V_1 - u) + (V_1 - u)\cos\phi] \times u}{V_1^2}$$

$$\boxed{\eta_h = \frac{2(V_1 - u)[1 + \cos\phi]u}{V_1^2}} \quad \text{—— (3)}$$

The efficiency will be maximum for a given value of V_1 when

$$\frac{d}{du}(\eta_h) = 0 \quad \text{or} \quad \frac{d}{du}\left[\frac{2u(V_1-u)(1+\cos\phi)}{V_1^2}\right] = 0$$

or $$\frac{(1+\cos\phi)}{V_1^2}\frac{d}{du}(2uV_1 - 2u^2) = 0$$

or $$\frac{d}{du}(2uV_1 - 2u^2) = 0 \qquad \left(\because \frac{1+\cos\phi}{V_1^2} = 0\right)$$

$$\Rightarrow \quad 2V_1 - 4u = 0$$

$$\Rightarrow \quad \cancel{2}V_1 = \overset{2}{\cancel{4}}u$$

$$\Rightarrow \quad \boxed{u = \frac{V}{2}}$$

Hence the hydraulic efficiency of a pelton wheel will be maximum when the velocity of the wheel is half the velocity of the jet of water at inlet.

$$\therefore \text{Max. } \eta_h = \frac{2\left(V_1 - \frac{V_1}{2}\right)(1+\cos\phi) \times \frac{V_1}{2}}{V_1^2}$$

$$= \frac{2 \times \left(\frac{2V_1 - V_1}{2}\right)(1+\cos\phi)\frac{V_1}{2}}{V_1^2}$$

$$\boxed{\text{Max } \eta_h = \frac{(1+\cos\phi)}{2}}$$

Scanned with CamScanner

Points to Remember for Pelton Wheel:

(i) Vel of the jet at inlet $V_1 = C_v\sqrt{2gH}$

where C_v = Co-efficiecy of Velocity = 0.98 or 0.99

H = Net head

(ii) ϕ = speed ratio varies from 0.43 to 0.48

(iii) The angle of deflection of the jet through buckets is taken as 165°.
if no angle of deflection is given

(iv) $u = \frac{\pi D N}{60}$ or $D = \frac{60u}{\pi N}$

where D = mean diameter or pitch diameter of Pelton Wheel

(v) Jet Ratio: It is defined as the ratio of the pitch diameter (D) of the pelton wheel to the diameter of jet (d)

$$m = \frac{D}{d} = \frac{\text{Pitch diameter of the Pelton Wheel}}{\text{Diameter of jet}}$$

(vi) Number of buckets on a runner is given by

$$Z = 15 + \frac{D}{2d} = 15 + 0.5m$$

Where m = Jet ratio

(vii) Number of jets = $\frac{\text{Total rate of flow through the turbine}}{\text{Rate of flow of water through a single jet}}$

Difference between Impulse and Reaction Turbine:

Impulse Turbine	Reaction Turbine
1. If at the inlet of the turbine the energy available is only kinetic energy, the turbine is known as impulse turbine	1. If at inlet of the turbine, the water possesses kinetic energy as well as pressure energy the turbine is known as reaction turbine.
2. This type of turbine consists of moving blades and nozzles	2. Reaction turbine consists of fixed blades that act as a moving blades and nozzle.
3. Efficiency is low	3. Efficiency is high.
4. It occupies less space per unit power.	4. It occupies more space per unit power.
5. The blades are symmetrical.	5. The blades are not symmetrical.
6. Maintenance is easy here	6. Maintenance of these turbine is not easy here.
7. There is no draft tube here.	7. In reaction turbine there is Draft tube.
8. The unit is installed above the tail race.	8. The unit is installed below the tailrace that means completely submerged in water.
9. It operates at higher water head	9. It operates at lower or medium water head.
10. An example of impulse turbine is pelton wheel turbine	10. An example of reaction turbine is the Kaplan Turbine and Francis Turbine.

Q A Pelton wheel has a mean bucket speed of 10 m/s with a jet of water flowing at the rate of 700 litre/s under a head of 30 m. The buckets deflect the jet through an angle of 160°. Calculate the power given by water to the runner and hydraulic efficiency of the turbine. Assume co-efficient of velocity as 0.98.

Given

Speed of bucket $u = u_1 = u_2 = 10 m/s$

Discharge $Q = 700 \, lt/s = 0.7 \, m^3/s$

Head of water $H = 30m$

Angle of deflection $= 160°$

$\therefore$ Angle $\phi = 180 - 160 = 20°$

Co-efficient of velocity $C_v = 0.98$

Velocity of jet $V_1 = C_v \sqrt{2gH} = 0.98 \sqrt{2 \times 9.81 \times 30}$

$= 23.77 \, m/s$

$V_{r_1} = V_1 - u_1 = 23.77 - 10 = 13.77 \, m/s$

$V_{w_1} = V_1 = 23.77 \, m/s$

Outlet velocity triangle $V_{r_2} = V_{r_1} = 13.77 m/s$

$V_{w_2} = V_{r_2} \cos\phi - u_2$

$= 13.77 \cos 20 - 10$

$= 2.94 \, m/s$

Scanned with CamScanner

Work done by the jet per second on the runner $= \rho a V_1 [V_{w_1} + V_{w_2}] u$

$= 1000 \times 0.7 [23.77 + 2.94] \times 10$

$= 186970$ Nm/s $\quad (\because aV_1 = Q = 0.7\, m^3/s)$

$\therefore$ Power given to turbine $= \frac{186970}{1000} = 186.97$ kW

Hydraulic efficiency of the turbine $(\eta_h) = \frac{2[V_{w_1} + V_{w_2}]u}{V_1^2}$

$= \frac{2(23.77 + 2.94) \times 10}{23.77 \times 23.77}$

$= 0.9454$

$= 94.54\%$

Q. A pelton wheel is to be designed for the following specifications:
Shaft power = 11772 kW ; Head = 380m ; speed = 750 r.p.m ; Overall efficiency = 86%
Jet diameter is not to exceed one-sixth of the wheel diameter. Determine
(i) The wheel diameter (ii) The number of jets required
(iii) Diameter of the jet Take $K_{v_1} = 0.985$, and $K_{u_1} = 0.45$
$C_v = 0.985$ and speed ratio $= 0.45$

Sol^n^

Given: Shaft power $S.P = 11772$ kW

Head $H = 380$ m

Speed $N = 750$ r.p.m

Overall efficiency $\eta_o = 86\%$ or 0.86

Ratio of jet dia to wheel dia. $\frac{d}{D} = \frac{1}{6}$

Co-efficient of velocity $K_v = C_v = 0.985$

Speed ratio $K_{u_1} = 0.45$

Velocity of jet $V_1 = C_v\sqrt{2gH} = 0.985\sqrt{2\times9.81\times380} = 85.05\,m/s$

Velocity of Wheel $u = u_1 = u_2$

$u = \text{speed ratio} \times \sqrt{2gH} = 0.45\times\sqrt{2\times9.81\times380}$

$= 38.85\,m/s$

$$u = \frac{\pi D N}{60} = \frac{\pi D\,750}{60}$$

$$\Rightarrow 38.85 = \frac{\pi D 750}{60}$$

$$\Rightarrow D = \frac{38.85\times60}{\pi\times750} = 0.989\ m$$

As $\frac{d}{D} = \frac{1}{6}$

$\therefore$ Dia of jet $d = \frac{1}{6}\times D = \frac{0.989}{6} = 0.165\ m$

Discharge of one jet (q) = Area of jet × Vel. of jet

$= \frac{\pi}{4}d^2\times V_1 = \frac{\pi}{4}\times(0.165)^2\times85.05\ m^3/s$

$= 1.818\ m^3/s$

$$\eta_o = \frac{S.P}{W.P} = \frac{11772}{\frac{\rho g Q H}{1000}}$$

$$\Rightarrow 0.86 = \frac{11772\times\cancel{1000}}{\cancel{1000}\times9.81\times Q\times380}$$ (where Q = Total discharge)

$\therefore$ Total discharge $Q = \frac{11772\cancel{00}}{0.86\times9.81\times380} = 3.672\ m^3/s$

$\therefore$ No. of jets $= \dfrac{\text{Total discharge}}{\text{Discharge of one jet}} = \dfrac{Q}{q} = \dfrac{3.672}{1.818}$

$= 2$ jets

Francis Turbine :

- The inward flow reaction turbine having radial discharge at outlet is known as Francis Turbine.
- It is after the ~~as~~ name of J. B Francis an American Engineer.
- In morden Francis Turbine, the water enters the runner of turrine in the radial direction at outlet and leaves in the axial direction at the inlet of ~~runne~~ runner.
- Thus Morden Francis Turbine is a mixed flow type turbine.

As in case of Francis turbine the discharge is radial at outlet

$\therefore$ Whril velocity at outlet $= V_{w_2} = 0$ and $\beta = 90°$

Work done by water on the runner per second $= \rho Q \left[V_{w_1} u_1 \right]$

Work done per second per unit weight of water striking/s $= \frac{1}{g} \left[V_{w_1} u_1 \right]$

Hydraulic efficiency $\eta_h = \dfrac{V_{w_1} u_1}{gH}$

Scanned with CamScanner

Important Relations for Francis Turbine:

1. The ratio of width of wheel to its diameter is given as $n = \frac{B_1}{D_1}$

   n varies from 0.10 to 0.40.

2. Flow ratio $= \frac{V_{f_1}}{\sqrt{2gH}}$ varies from 0.15 to 0.30.

3. Speed ratio $= \frac{u_1}{\sqrt{2gH}}$ varies from 0.6 to 0.9.

Q A Francis turbine with an overall efficiency of 75% is required to produce 148.25 kW power. It is working under a head of 7.62m. The peripheral velocity $= 0.26\sqrt{2gH}$ and the radial velocity of flow at inlet is $0.96\sqrt{2gH}$. The wheels runs at 150 rpm and the hydraulic losses in the turbine are 22% of the available energy. Assuming radial discharge, determine:

(i) The guide blade angle (ii) The wheel vane angle at inlet
(iii) Diameter of the wheel at inlet (iv) Width of the wheel at inlet

Solⁿ:

Given:

Overall Efficiency $\eta_o = 75\% = 0.75$

Power produced $S.P = 148.25$ kW

Head $H = 7.62$ m

Peripheral Velocity $u_1 = 0.26\sqrt{2gH} = 0.26\sqrt{2 \times 9.81 \times 7.62} = 3.179$ m/s

Velocity of flow at inlet $V_{f_1} = 0.96\sqrt{2gH} = 0.96\sqrt{2 \times 9.81 \times 7.62} = 11.738$ m/s

Speed $N = 150$ r.p.m

Hydraulic losses = 22% of available energy

Discharge at outlet = Radial

$$V_{w_2} = 0 \quad \text{and} \quad V_{f_2} = V_2 \;, \beta = 90^\circ$$

$$\text{Hydraulic efficiency } (\eta_h) = \frac{\text{Total head at inlet - Hydraulic loss}}{\text{Head at inlet}}$$

$$= \frac{H - 0.22H}{H} = \frac{0.78H}{H} = 0.78$$

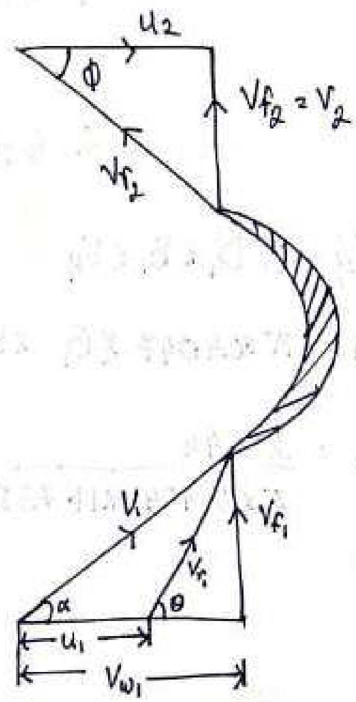
$$\eta_h = \frac{V_{w_1} u_1}{gH}$$

$$\therefore \; 0.78 = \frac{V_{w_1} u_1}{gH}$$

$$\therefore \; V_{w_1} = \frac{0.78 \times g \times H}{u_1}$$

$$= \frac{0.78 \times 9.81 \times 7.62}{31.79}$$

$$= 18.34 \text{ m/s}$$



(i) The guide blade angle i.e α.

$$\tan\alpha = \frac{V_{f_1}}{V_{w_1}} = \frac{11.738}{18.34} = 0.64$$

$$\Rightarrow \alpha = \tan^{-1} 0.64 = 32.619$$

(ii) The wheel vane angle at inlet i.e θ

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{11.738}{18.34 - 3.179} = 0.774$$

$$\theta = \tan^{-1} 0.774 = 37.74^\circ$$

(iii) Diameter of wheel at inlet (D_1)

$$u_1 = \frac{\pi D_1 N}{60}$$

$$D_1 = \frac{60 \times u_1}{\pi \times N} = \frac{60 \times 3.179}{\pi \times 50} = 0.4047 m$$

(iv) Width of the wheel at inlet (B_1)

$$\eta_o = \frac{S.P}{W.P} = \frac{148.25}{W.P}$$

$$\Rightarrow W.P = \frac{WH}{1000} = \frac{\rho \times g \times Q \times H}{1000} = \frac{1000 \times 9.81 \times Q \times 7.62}{1000}$$

$$\therefore \quad \eta_o = \frac{148.25}{\frac{\cancel{1000} \times 9.81 \times Q \times 7.62}{\cancel{1000}}} = \frac{148.25}{9.81 \times Q \times 7.62}$$

$$= 2.644 m^3/s$$

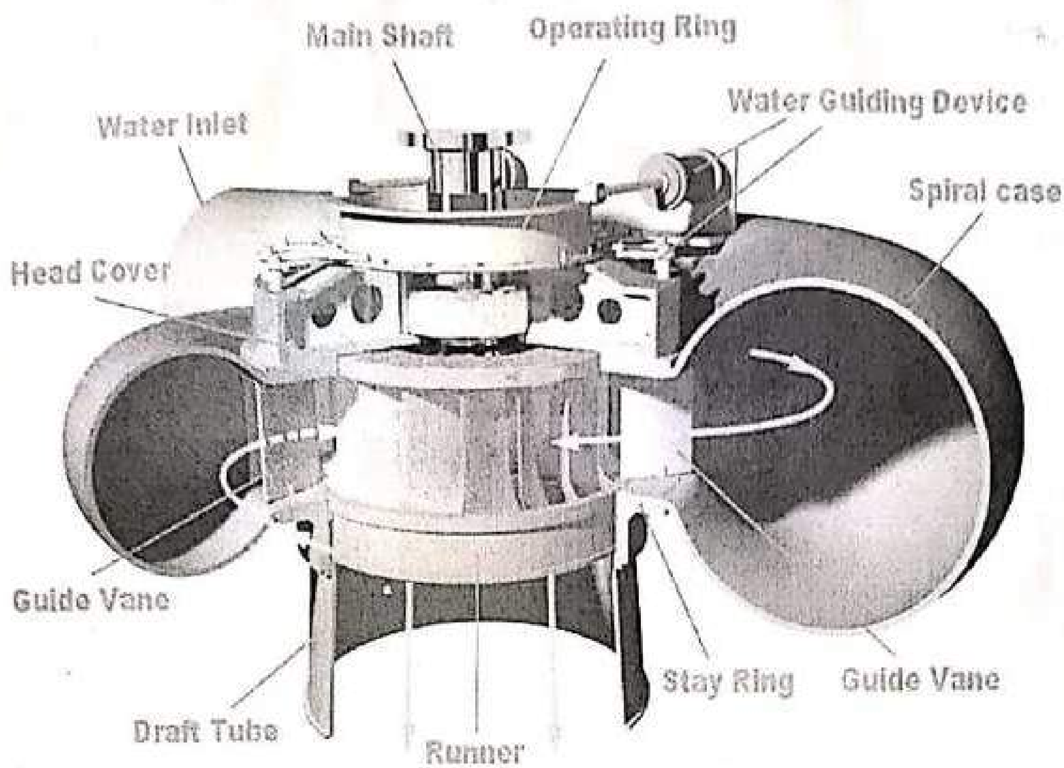
$$Q = \pi D_1 \times B_1 \times V_{f_1}$$

$$\Rightarrow 2.644 = \pi \times 0.4047 \times B_1 \times 11.738$$

$$\Rightarrow B_1 = \frac{2.644}{\pi \times 0.4047 \times 11.738} = 0.177 m$$

Main Components of Francis Turbine:

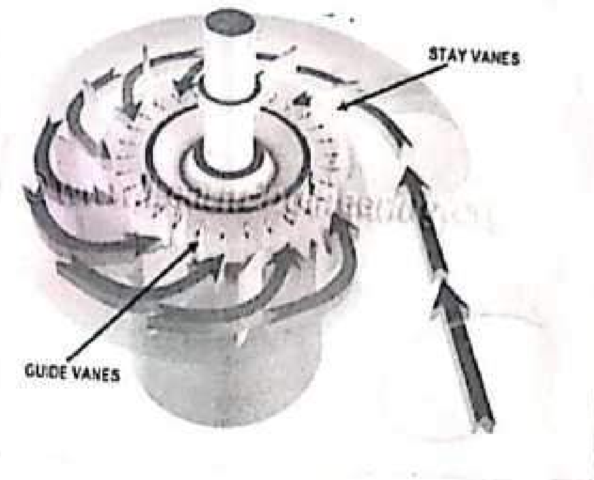
1. Sprial Casing: Sprial Casing is the inlet medium of water to the turbine. The water flowing from the reservoir or dam is made to pass through this pipe with high pressure.



Francis Turbine

The blades of the turbine are circularly placed which means the water striking the turbines blades should flow in the circular axis for efficient striking. But due to circular movement of the water it looses its pressure & to mantain pressure drop the diameter of casing is gradually reduced.

2. Stay Vanes: Stay vanes and guide vanes guides the water to the runner blades. Stay vanes remain stationary at their position and reduces the swirling of water due to the radial flow as it enters the runner blades.



Stay Vanes and guide Vanes of Francis Turbine.

3. Guide Vanes: Guide vanes are stationary, they change their angle as per the requirement to control the angle of striking of water to turbine blades to increase the efficiency and also regulate the flow rate water into the runner blades.

4. Runner Blades: These are arranged at the centre of the turbine. where the water hits and the tangential power of impact causes the shaft to turn for generating torque.

5. Draft Tube: The water at exit cannot be directly discharged to the tail race. A tube or pipe of gradually ~~increase~~ increasing area is used for discharging water from the exit of the turbine to the tail race known as draft tube.

Scanned with CamScanner

Working of Francis Turbine:

- The water enters into the turbine through volute casing and ~~thats~~ then to the guide blades and stationary blades.
- The volute casing keeps in reducing diameter to mantain the flow pressure.
- The stationary blades remain fixed at their position, which eliminates the water vortices.
- The guide blade's angle determines the angle of the water on the impeller blades ~~are ground~~ and ensures the performance of turbine.
- The water flows through the guide blades or guide vanes and is directed towards the runner blades at optimum angles.
- Since the water crosses the precisely curved blades of the runner the water is diverted somewhat sideways to create "Whirl motion".
- The water is then deflected in the axial direction to exit or draft tube to the tail race.

A Francis turbine with an overall efficiency of 75% is required to produce 148.25 KW power. It is working under a head of 7.62 m. The Peripheral velocity = $0.26\sqrt{2gH}$ and the radial velocity of flow at inlet is $0.96\sqrt{2gH}$. The wheel runs at 150 r.p.m and the hydraulic losses in the turbine are 22% of the available energy. Assuming radial ~~distance~~ discharge, determine:

(i) The guide blade angle (ii) The wheel vane angle at inlet
(iii) Diameter of the wheel at inlet (iv) Width of the wheel at inlet.

Solution:

Given:

Overall efficiency $\eta_o = 75\% = 0.75$

Power produced $S.P = 148.25\ KW$

Head $H = 7.62\,m$

Peripheral Velocity $u_1 = 0.26\sqrt{2gH} = 0.26 \times \sqrt{2 \times 9.81 \times 7.62} = 3.179\,m/s$

Velocity of flow at inlet $V_{f_1} = 0.96\sqrt{2gH} = \cancel{0.2}\ 0.96\sqrt{2 \times 9.81 \times 7.62} = 11.738\,m/s$

Speed $N = 150\ r.p.m$

Hydraulic losses $= 22\%$ of available energy

As discharge at outlet is radial

$\Rightarrow V_{w_2} = 0 \qquad V_{f_2} = V_2$

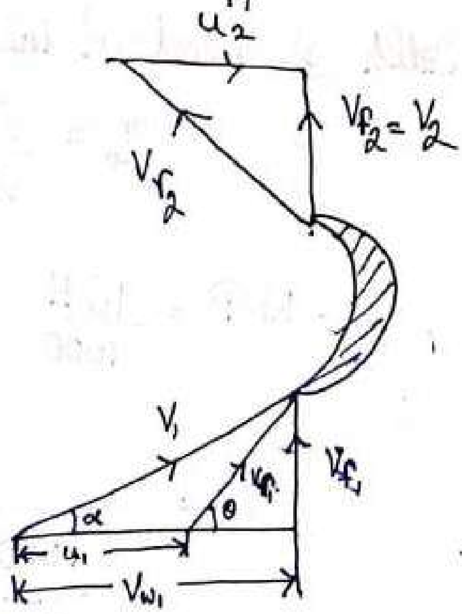
Scanned with CamScanner

Hydraulic efficiency is given as

$$\eta_h = \frac{\text{Total head at inlet} - \text{Hydraulic Losses}}{\text{Head at inlet}}$$

$$\eta_h = \frac{H - 0.22H}{H} = \frac{0.78H}{H}$$

But we know $\eta_h = \frac{V_w u_1}{gH}$



$$\Rightarrow \frac{V_{w_1} u_1}{gH} = 0.78$$

$$\Rightarrow V_{w_1} = \frac{0.78 \times g \times H}{u_1}$$

$$= \frac{0.78 \times 9.81 \times 7.62}{3.179} = 18.34 \text{m/s}$$

(i) Guide blade angle i.e α

$$\tan\alpha = \frac{V_{f_1}}{V_{w_1}} = \frac{11.738}{18.34} = 0.64$$

(ii) The Wheel Vane angle at inlet i.e θ

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{11.738}{18.34 - 3.179} = 0.774$$

$$\theta = \tan^{-1} 0.774 = 37.74$$

(iii) Diameter of wheel at inlet (D_1)

$$u_1 = \frac{\pi D_1 N}{60}$$

$$D_1 = \frac{60 \times u_1}{\pi N} = \frac{60 \times 3.179}{\pi \times 50} = 0.4047\,m$$

(iv) Width of Wheel at inlet (B_1)

$$\eta_o = \frac{S.P}{W.P} = \frac{148.25}{W.P}$$

$$W.P = \frac{WH}{1000} = \frac{\rho g Q H}{1000} = \frac{1000 \times 9.81 \times Q \times 7.62}{1000}$$

$$\eta_o = \frac{148.25}{\frac{\cancel{1000} \times 9.81 \times Q \times 7.62}{\cancel{1000}}} = \frac{148.25}{9.81 \times Q \times 7.62}$$

$$Q = \frac{148.25}{9.81 \times 7.62 \times 0.75} = 2.644\,m^3/s$$

$$Q = \pi D_1 B_1 V_{f_1}$$

$$\Rightarrow 2.644 = \pi \times 0.4047 \times B_1 \times 11.738$$

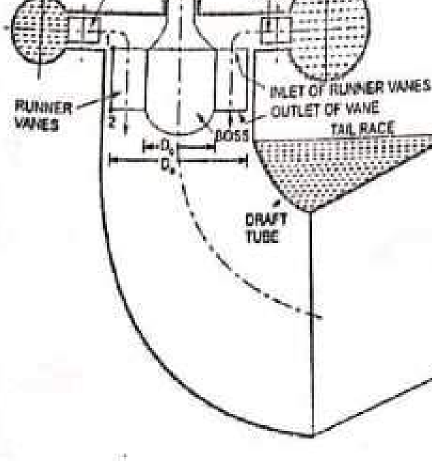
$$\Rightarrow B_1 = \frac{2.644}{\pi \times 0.4047 \times 11.738} = 0.177\,m$$

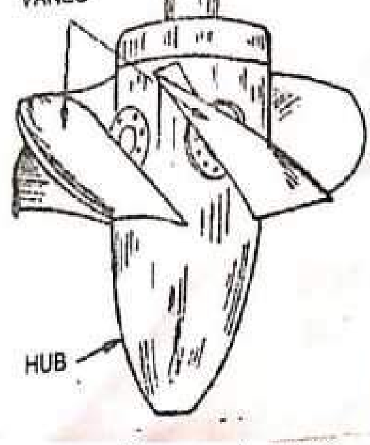
Scanned with CamScanner

Kaplan Turbine: Kaplan turbine is axial flow reaction turbine in which water flows parallel to the axis of rotation of the shaft and head at the inlet of the turbine is the sum of pressure energy and kinetic energy and during the flow of water through runner a part of pressure energy is converted into kinetic energy.

- Kaplan turbine was named after the name of V Kaplan an Austrian Engineer.
- The axial flow reaction have vertical shaft and the lower end of the shaft is made larger which is known as 'hub' or 'boss'
- The vanes on the hub are adjustable in kaplan turbine.
- Kaplan Turbine is suitable for large quantity of water low
- The main parts of Kaplan turbine are:

1. Scroll Casing: It is a sprial type of casing with reducing cross-sectional area. The water from the penstock enters the scroll cover and then into the guide vane, where the water passes 90° and flows axially through the runner.





Guide Vane: It is the only controlled part of the entire turbine, which opens and closes depending on the demand for electricity required. when greater power generation is required it opens wider to allow more water to hit the rotor blades and Vice-versa happens for low power output.

Hub with vanes or runner of the turbine: The runner of this turbine has a large 'boss' or hub with its vanes mounted and the vanes of the runner are adjustable to an optimum angle of attack for maximum power output.

Draft Tube: A tube or pipe of a slowly growing area is used to discharge water from the exit of the turbine to the tailrace. Known as ~~drag~~ draft tube.

Working of Kaplan Turbine:

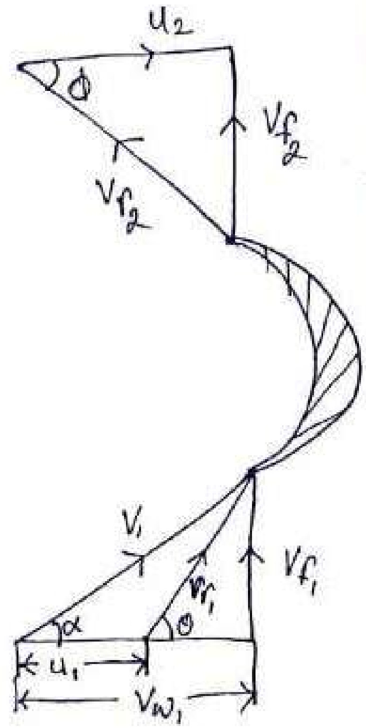
- The water is poured into the scroll casing before the penstock. The cross-section of the scroll casing decreases evenly to mantain water pressure.
- Then with scroll casing the guide vanes transport the water to the runner or vanes. The vanes or runer are adjustable to mantain optimal angle for the varying flow rates.
- From the runner blades water enters the draft tube, where the Kinetic and pressure energy of the turbine decreases.

- Kinetic energy is converted into pressure energy, which leads to increase water pressure and finally water discharge from the turbine through the tail race.
- The runner rotates the rotation shaft of the blade to which the runner blades are attached.
- This rotation of the Shaft is used for power generation.

Q A Kaplan turbine working under a head of 20 m develops 11772 kW Shaft power. The outer diameter of the runner is 3.5 m and hub diameter 1.75m. The guide blade angle at the extreme edge of the runner is ~~2.5m~~ 35°. The hydraulic and overall efficiencies of the turbine are 88% and 84% respectively. If the velocity of Whirl is zero at outlet, determine:

(i) Runner Vane angle at inlet and outlet at the extreme edge of the runner

(ii) Speed of turbine.



Given :

Head $H = 20m$

Shaft power $S.P = 11772\ kW$

Outer diameter of runner $D_o = 3.5m$

Hub diameter $D_b = 1.75m$

Guide blade angle $\alpha = 35°$

Hydraulic efficiency $\eta_h = 88\%$

overall efficiency $\eta_o = 84\%$

$$\eta_o = \frac{S.P}{W.P}$$

$$W.P = \frac{W.P}{1000} = \frac{\rho \times g \times Q \times H}{1000}$$

Scanned with CamScanner

$$\Rightarrow 0.84 = \frac{11772}{\frac{\rho \times g \times Q \times H}{1000}}$$

$$= \frac{11772 \times 1000}{1000 \times 9.81 \times Q \times 20} \qquad (\because \rho = 1000)$$

$$Q = \frac{11772 \times 1000}{0.84 \times 1000 \times 9.81 \times 20} = 71.428 \ m^3/s$$

$$Q = \frac{\pi}{4}(D_0^2 - D_b^2) \times V_{f_1}$$

$$\Rightarrow 71.428 = \frac{\pi}{4}(3.5^2 - 1.75^2) \times V_{f_1} = \frac{\pi}{4}(12.25 - 3.0625) V_{f_1}$$

$$\Rightarrow 71.428 = 7.216 \, V_{f_1}$$

$$\Rightarrow V_{f_1} = \frac{71.428}{7.216} = 9.9 \ m/s$$

inlet Velocity triangle, $\tan\alpha = \frac{V_{f_1}}{V_{w_1}}$

$$V_{w_1} = \frac{V_{f_1}}{\tan\alpha} = \frac{9.9}{\tan 35^\circ} = \frac{9.9}{0.7} = 14.4 \ m/s$$

Hydraulic Efficiency,

$$\eta_h = \frac{V_{w_1} u_1}{gH} \qquad (\because V_{w_2} = 0)$$

$$0.88 = \frac{14.14 \times u_1}{9.81 \times 20}$$

$$u_1 = \frac{0.88 \times 9.81 \times 20}{14.14} = 12.21 \ m/s$$

(i) Runner vane angles at inlet & outlet at the extreme edge of the runner

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{9.9}{14.14 - 12.21} = 5.13$$

$$\theta = \tan^{-1} 5.13 = 78.97^\circ$$

$u_1 = u_2 = 12.21$ m/s & $V_{f_1} = V_{f_2} = 9.9$ m/s

$\therefore$ From outlet velocity triangle, $\tan\phi = \frac{V_{f_2}}{u_2} = \frac{9.9}{12.21} = 0.811$

$$\phi = \tan^{-1} 0.811 = 39.035^\circ$$

(ii) Speed of turbine is given by $u_1 = u_2 = \frac{\pi D_o N}{60}$

$$12.21 = \frac{\pi \times 3.5 \times N}{60}$$

$$\Rightarrow N = \frac{60 \times 12.21}{\pi \times 3.50} = 66.63 \text{ rpm.}$$

Centrifugal Pumps

Pumps: The hydraulic machines which convert the mechanical energy into hydraulic energy are called pump.

Centrifugal Pump: If the mechanical energy is converted into pressure energy by means of centrifugal force acting on the fluid, the hydraulic machine is called centrifugal pump. The centrifugal pump acts as a reverse of an inward radial flow reaction turbine. Therefore the flow in centrifugal pumps is in the radial outward directions. It is used in places like agriculture, municipal (water & wastewater plant), industrial, power generation plants, petroleum, mining pharmaceutical etc.

Main parts of Centrifugal Pump:

The following are the main parts of a centrifugal pump:

1. Impeller
2. Casting
3. Suction pipe with a foot valve and a strainer
4. Delivery pipe.

Centrifugal Pumps

Pumps: The hydraulic machines which convert the mechanical energy into hydraulic energy are called pump.

Centrifugal Pump: If the mechanical energy is converted into pressure energy by means of centrifugal force acting on the fluid, the hydraulic machine is called centrifugal pump. The centrifugal pump acts as a reverse of an inward radial flow reaction turbine. Therefore the flow in centrifugal pumps is in the radial outward directions. It is used in places like agriculture, municipal (water & wastewater plant), industrial, power generation plants, petroleum, mining pharmaceutical etc.

Main parts of Centrifugal Pump:

The following are the main parts of a centrifugal pump:

1. Impeller
2. Casting
3. Suction pipe with a foot valve and a strainer
4. Delivery pipe.

1. Impeller: The rotating part of a centrifugal pump is called 'impeller'. It consists of a series of backward curved vanes. The impeller is mounted on a shaft which is connected to the shaft of an electric motor.

2. Casing: The casing of of centrifugal pump is similar to reaction turbine. It is an air-tight passage surrounding the impeller and is designed is such a that kinetic energy of the water discharged at the outlet of the impeller is converted into pressure energy before the water leaves the casing and enters the delivery pipe.

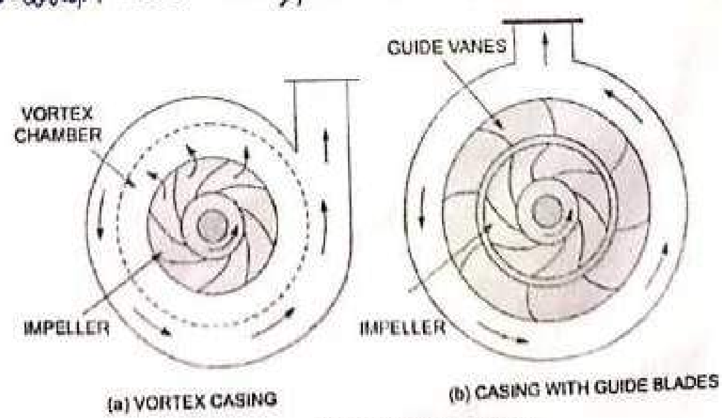
The following three types of the casing are commonly adopted:

(a) Volute Casing

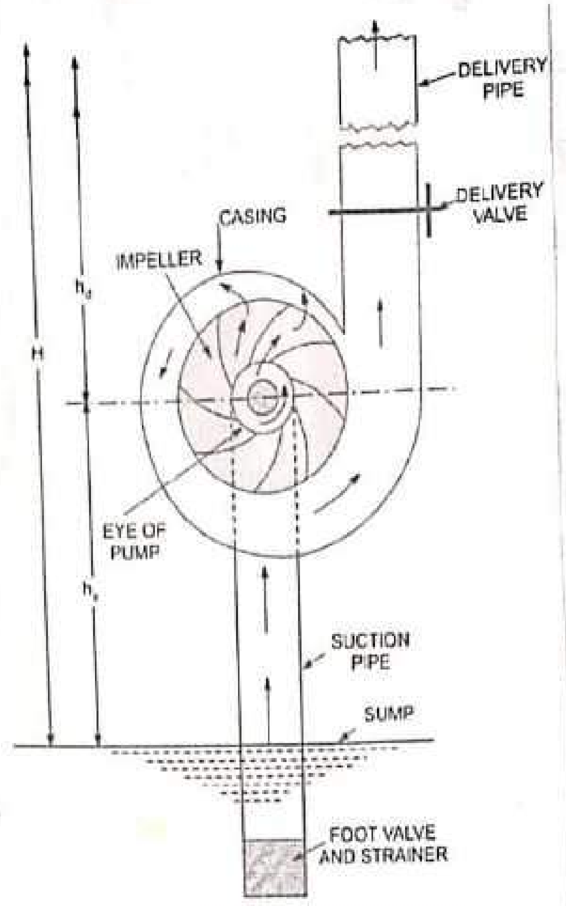
(b) Vortex Casing

(c) Casing with guide blades

(a) Volute Casing: Volute casing, which surrounds the impeller. It is spiral type in which area of flow increases gradually and velocity of flow decreases. The decrease in velocity increases the pressure of the water flowing through the casing.



Different types of casing.



Main parts of a centrifugal pump.

(b) Vortex Casing: If a circular chamber is introduced between the casing and the impeller as shown in figure 2.1 the casing is known as vortex casing. Because of circular chamber the loss of energy due to formation of eddies is reduced to a considerable extent. Hence efficiency is more as compared to volute casing.

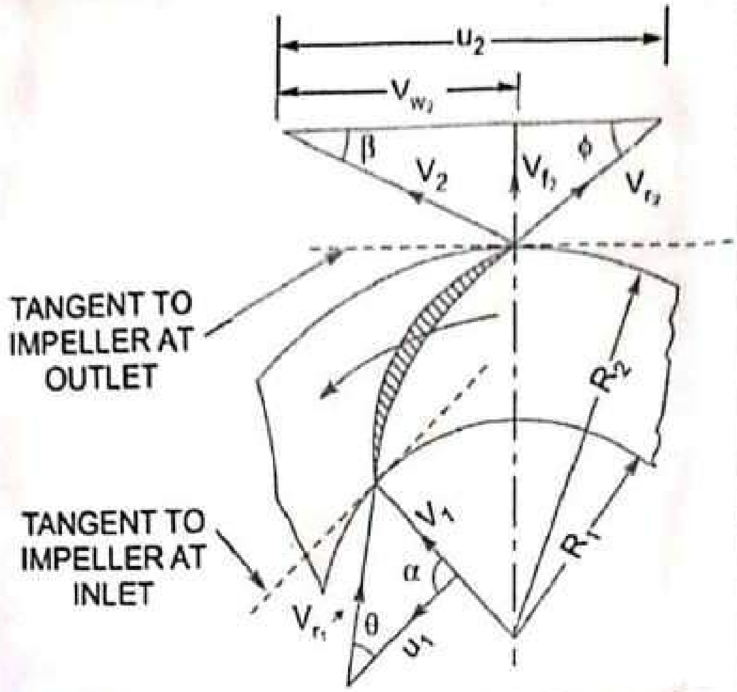
(c) Casing with Guide Blades: As shown in fig 2.1 in which the impeller is surrounded by a series of guide blade mounted on ring known as diffuser. The guide vanes are designed in such a way that the water from the impeller enters the guide vanes without stock.

3. Suction pipe with a foot valve and a stainer: A pipe whose one end is connected to the pump and the other end dip into water in a sump known as suction pipe. A foot valve which acts as a non-return valve and opens only upwards is fitted at the lower end of the suction pipe. A stainer is also fitted at the lower end of suction pipe.

4. Delivery Pipe: A pipe whose one end is connected to the outlet of the pump and other end delivers the water at a required height known as delivery pipe.

Working of Centrifugal Pump:

- A centrifugal pump uses a centrifugal force to pump the fluids ~~too~~ hence known as centrifugal pump.
- The mechanical power is given by electric motor to the impeller.
- The impeller directly connects with the electric motor through a shaft and reciprocates with the motion of the motor shaft.
- When the impeller starts rotating, a vacuum starts generating inside the impeller's eye. Due to this vacuum the water starts entering from the sump through the suction pipe to the impeller.
- As the water enters in the impeller eye, the water strikes the blades of the impeller.
- The impeller rotates the water radially and axially outward with the help of centrifugal force.
- Since the impeller is moving at high velocity it also rotates the water around it.
- The area of casing increases slowly in the direction of rotation, so as the water velocity decreases and pressure increases, the pressure at the outlet of the pump is maximized.
- ~~No~~ From the outlet of the pump the water goes through the delivery pipe to its intended location.



Let N = Speed of the impeller in rpm

D_1 = Diameter of impeller at inlet

u_1 = Tangential velocity of impeller at inlet $= \frac{\pi D_1 N}{60}$

D_2 = Diameter of impeller at outlet

u_2 = Tangential velocity of impeller at outlet $= \frac{\pi D_2 N}{60}$

V_1 = Absolute velocity of water at inlet

α = Angle made by absolute velocity (V_1) at inlet with the direction of motion of vane

θ = Angle made by relative velocity at inlet with the direction of motion of vane and V_2

V_{r_2}, β and ϕ the corresponding values at outlet.

As the water enters the impeller radially which means the absolute velocity of water at inlet is in the radial direction and hence angle $\alpha = 90°$ and $V_{w_1} = 0$

A centrifugal pump is the reverse of a ~~...~~ radially inward flow reaction turbine

$\therefore$ water striking per second $= \frac{1}{g}\left[V_{w_1} u_1 - V_{w_2} u_2\right]$

$\therefore$ Work done by the impeller on the water per second per unit weight of water striking per second $= -$ [work done in case of turbine]

$$= -\left[\frac{1}{g}\left(V_{w_1} u_1 - V_{w_2} u_2\right)\right]$$

$$= \frac{1}{g}\left[V_{w_2} u_2 - V_{w_1} u_1\right]$$

$$= \frac{1}{g} V_{w_2} u_2 \qquad (\because V_{w_1} = 0)$$

Work done by impeller on water per second $= \frac{W}{g} V_{w_2} u_2$

where $W =$ Weight of water $= \rho g Q$

$Q =$ Volume of water

$Q =$ Area $\times$ Velocity of flow $= \pi D_1 B_1 V_{f_1}$

$= \pi D_2 B_2 V_{f_2}$

where B_1 and B_2 are width of impeller at inlet and outlet.

Definations of Heads and efficiencies of a Centrifugal Pump

1. **Suction Head (h_s)**: It is the vertical height of the centre line of the centrifugal pump ~~are~~ above the water surface in the tank or pump from which the water is to be lifted.

2. **Delivery Head (hd)**: The vertical distance between the centre lines of the pump and the water surface in the tank to which water is delivered known as delivery head.

3. **Static Head** (H_s) The sum of suction head and delivery head is known as static head.

$$\boxed{H_s = h_s + h_d}$$

4. **Manometric Head (H_m)**: The manometric head is defined as the head against which a centrifugal pump has to work.

(a) H_m = Head imparted by the impeller to water - Loss of head in the pump

$$\boxed{H_m = \frac{V_{w_2} u_2}{g} - \text{Loss of head in impeller + casing}}$$

(b) H_m = Total head at outlet of the pump - Total head at the inlet of Pump

$$\boxed{H_m = \left(\frac{P_o}{\rho g} + \frac{V_o^2}{2g} + Z_o\right) - \left(\frac{P_i}{\rho g} + \frac{V_i^2}{2g} + Z_i\right)}$$

where $\frac{P_o}{\rho g}$ = Pressure head at outlet of the pump - h_d

$\frac{V_o^2}{2g}$ = Velocity head at outlet of the pump

$\frac{V_d^2}{2g}$ = Velocity head at delivery pipe

Z_o = Vertical height of the ~~exit~~ outlet of the pump from datum line

$\frac{P_i}{\rho g}, \frac{V_i^2}{2g}, Z_i$ = Corresponding values of pressure head, velocity head and datum head at the inlet of the pump

(c) $\boxed{H_m = h_s + h_d + h_{f_s} + h_{f_d} + \frac{V_d^2}{2g}}$

where h_s = Suction head , h_d = Delivery head

h_{f_s} = Frictional head loss in Suction pipe , h_{f_d} = Frictional head loss in delivery pipe

V_d = Velocity of water in delivery pipe

5. Efficiencies of Centrifugal Pump:

(a) Manometric efficiency, η_{man} (b) Mechanical efficiency, η_m

(c) Overall efficiency, η_o

(a) Manometric Efficiency (η_{man}): The ratio of the manometric head to the head imparted by the impeller to the water is known as manometric efficiency.

$$\eta_{man} = \frac{\text{Manometric head}}{\text{Head imparted by impeller to water}}$$

$$\eta_{man} = \frac{H_m}{\left(\frac{V_{w_2} u_2}{g}\right)} = \frac{g H_m}{V_{w_2} u_2}$$

$$\boxed{\eta_{man} = \frac{g H_m}{V_{w_2} u_2}}$$

The ratio of the power given to water at outlet of the pump to the pump to the power available at the impeller is known as manometric efficiency.

$$\text{Power given to water at outlet of pump} = \frac{W H_m}{1000} \text{ kW}$$

$$\text{Power at the impeller} = \frac{\text{Work done by impeller per sec}}{1000} \text{ kW}$$

$$= \frac{W}{g} \times \frac{V_{w_2} u_2}{1000}$$

$$\eta_{man} = \frac{\frac{\cancel{W} H_m}{\cancel{1000}}}{\frac{\cancel{W}}{g} \times \frac{V_{w_2} u_2}{\cancel{1000}}} = \frac{g H_m}{V_{w_2} u_2}$$

(b) Mechanical Efficiency (η_m): The power at the shaft of the Centrifugal pump is more than the power available at the impeller of the pump. The ratio of power available at the impeller to the power at the shaft of the centrifugal pump is known as mechanical efficiency.

$$\eta_m = \frac{\text{Power at the impeller}}{\text{Power at the shaft}}$$

The power at the impeller in kW = $\frac{\text{work done by impeller per sec}}{1000}$

$$= \frac{W}{g} \times \frac{V_{w_2} u_2}{1000}$$

$$\boxed{\eta_m = \frac{\frac{W}{g}\left(\frac{V_{w_2} u_2}{1000}\right)}{S.P}}$$

(c) Overall Efficiency (η_o): It is defined as ratio of power output of the pump to the power input to the pump.

power output of the pump in kW = $\frac{\text{weight of water lifted} \times H_m}{1000}$

$$= \frac{W H_m}{1000}$$

Power input to the pump = Power supplied by the electric motor

= S.P of the pump

$$\boxed{\eta_o = \frac{\frac{W H_m}{1000}}{S.P}}$$

$$\boxed{\eta_o = \eta_{man} \times \eta_m}$$

Scanned with CamScanner

Q. The internal and external diameters of the ~~impeller~~ impeller of a Centrifugal pump are 200mm and 400mm respectively. The pump is running at 1200 rpm. The vane angle of the impeller at inlet and outlet are 20° and 30° respectively. The water enters the impeller radially and velocity of flow is constant. Determine the work done by the impeller per unit weight of water.

Sol: Given:

Internal diameter of impeller, $D_1 = 200mm = 0.2m$

External diameter of impeller, $D_2 = 400\,mm = 0.4\,m$

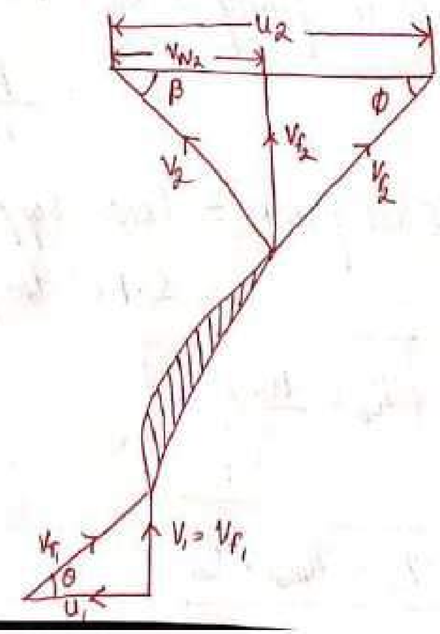
Speed $N = 1200\,rpm$

Vane angle at inlet, $\theta = 20°$

Vane angle at outlet, $\phi = 30°$

Water enters radially $\Rightarrow \alpha = 90°$ & $V_{w_1} = 0$

Velocity of flow $V_{f_1} = V_{f_2}$



Tangential velocity of impeller at inlet and outlet are

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.2 \times 1200}{60} = 12.56 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.4 \times 1200}{60} = 25.13 \text{ m/s}$$

From inlet velocity triangle $\tan\theta = \frac{V_{f_1}}{u_1} = \frac{V_{f_1}}{12.56}$

$$V_{f_1} = 12.56 \tan\theta = 12.56 \times \tan 20^\circ = 4.57 \text{ m/s}$$

$$V_{f_2} = V_{f_1} = 4.57 \text{ m/s}$$

Outlet velocity triangle, $\tan\phi = \frac{V_{f_2}}{U_2 - V_{w_2}} = \frac{4.57}{\tan 30^\circ} = 7.915$

$$V_{w_2} = 25.13 - 7.915 = 17.215 \text{ m/s}$$

The work done by impeller per kg of water per second

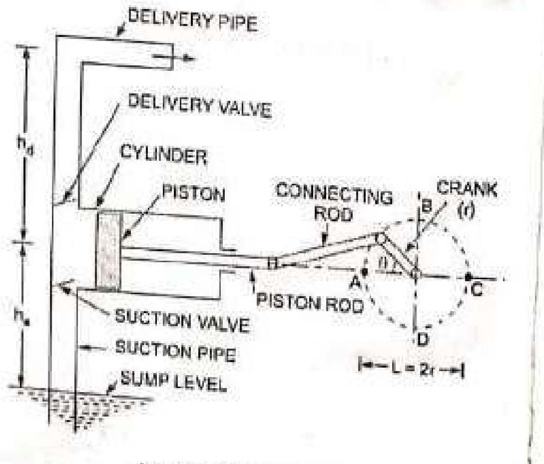
$$= \frac{1}{g} V_{w_2} U_2 = \frac{17.215 \times 25.13}{9.81} = 44.1 \text{ Nm/N}$$

Reciprocating Pump

Reciprocating Pump: If the mechanical energy is converted into hydraulic energy (or pressure energy) by Sucking the liquid into a cylinder in which a piston is reciprocating (moving backward and forwards) which exerts the thrust on the liquid and increases its hydraulic Energy (Pressure energy) the pump is known as reciprocating Pump.

Main Parts of a Reciprocating Pump: The following are main parts of reciprocating Pump.

1. A Cylinder with a piston, piston rod, connecting rod + crank.
2. Suction Pipe.
3. Delivery Pipe.
4. Suction Valve.
5. Delivery Valve.



Main parts of a reciprocating pump.

Working of a Reciprocating Pump:

- In fig 3.1 shows a single acting reciprocating pump which consists of a piston which moves forward and backward in a close fitting cylinder.
- The movement of the piston is obtained by connecting the piston rod to crank by means of connecting rod.
- The crank is rotated by means of an electric mortor.
- Suction and Delivery pipes are attached with suction valve & delivery valves are connected to the cylinder.
- The suction and delivery valves are non-return valves, which allows the water to flow in one direction olny.
- As the crank starts rotating from A to C (i.e from $\theta = 0°$ to $\theta = 180°$) the piston starts moving towards right in the cylinder.
- Due to the movement of the piston towards right creates a partial vacuum in the cylinder.
- But on the surface of the liquid in the sump atmospheric pressure is acting which is more than the pressure inside the cylinder.
- Thus the liquid is forced in the suction pipe from the sump.
- Hence the suction valve opens and liquid enters the cylinder.
- When crank is rotating from C to A (i.e from $\theta = 180°$ to $\theta = 360°$) the piston from its extreme right position starts moving towards the left in the cylinder.
- The movement of the piston towards left increases the pressure of the liquid inside the cylinder more than atmospheric pressure.
- Hence suction valve closes and delivery valve opens.
- The liquid is forced into the delivery pipe and is raised to a required height.

Discharge Through a Reciprocating Pump:

Let us consider a Single acting reciprocating Pump as shown in fig 3.1

Let D = Diameter of the Cylinder.

A = Cross-sectional area of the piston or cylinder

$= \frac{\pi}{4} D^2$

r = Radius of crank.

N = r.p.m of the crank.

L = Length of the Stroke = $2r$

h_s = Height of the axis of the cylinder from water Surface in Sump

h_d = Height of delivery outlet above the cylinder axis (known as delivery head)

Volume of water delivered in one revolution or

Discharge of water in one revolution = Area x Length of stroke

$= A \times L$

Number of revolution per second = $\frac{N}{60}$

∴ Discharge of the pump per second,

Q = Discharge in one revolution x No. of revolution per sec

$= A \times L \times \frac{N}{60}$

$= \frac{ALN}{60}$

Weight of water delivered per second,

$$W = \rho g Q = \frac{\rho g A L N}{60}$$

Work done by Reciprocating Pump:

Work done by reciprocating Pump per second = Weight lifted per second × Total height through which water is lifted

$$= W \times (h_s + h_d) \qquad \text{— (1)}$$

Where $(h_s + h_d)$ = Total height through which water is lifted

$$W \text{ (weight)} = \frac{\rho g \times A L N}{60}$$

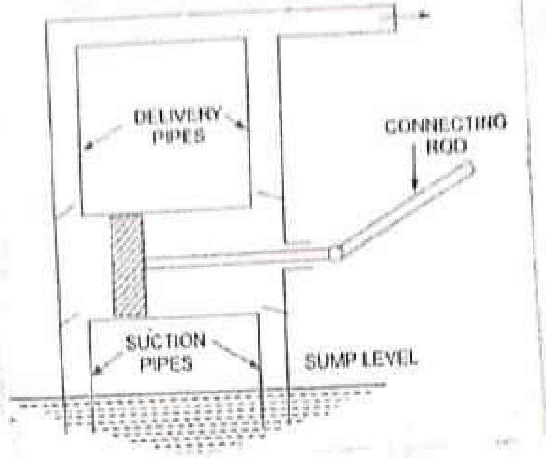
Substituting the value of W in equation (1) we have

$$\text{Work done per second} = \frac{\rho g \times A L N}{60} (h_s + h_d)$$

∴ Power required to drive the pump in kW

$$P = \frac{\text{Work done per sec}}{1000} = \frac{\rho g \times A L N \times (h_s + h_d)}{60 \times 1000}$$

$$= \frac{\rho g \times A L N \times (h_s + h_d)}{60000} \text{ kW}$$



Discharge, work done and Power required to drive a double-acting Pump

- In double-acting reciprocating pump, the water is acting on both sides of the piston as shown in the figure given above

- Hence two suction pipes and two delivery pipes for double-acting pumps are required.

- When there is suction stroke on one side of the piston, there is at the same time a delivery stroke on the other side of the piston.

- In one complete revolution of the crank there are two delivery strokes and water is delivered to the pipes by the pump during these two delivery strokes.

Let D = Diameter of the piston — Discharge through a double acting Reciprocating Pump

d = Diameter of the piston rod

$\therefore$ Area on one side of the Piston $(A) = \frac{\pi}{4} D^2$

Area on the otherside of the piston $(A_1) = \frac{\pi}{4} D^2 - \frac{\pi}{4} d^2$ (where piston rod is connected to the Piston)

$\therefore$ Volume of water delivered in one revolution of crank

$$= A \times \text{Length of stroke} + A_1 \times \text{Length of stroke}$$

$$= AL + A_1 L = (A + A_1)L = \left[\frac{\pi}{4} D^2 + \frac{\pi}{4}(D^2 - d^2)\right] L$$

$\therefore$ Discharge of Pump per second = Volume of water delivered in one revolution $\times$ No. of revolution per second

$$= \left[\frac{\pi}{4} D^2 + \frac{\pi}{4}(D^2 - d^2)\right] \times L \times \frac{N}{60}$$

If 'd' the diameter of the piston rod is very small as compared to the diameter of the piston, then it can be neglected and discharged

Discharge of Pump per second $(Q) = \left(\frac{\pi}{4} D^2 + \frac{\pi}{4} D^2\right) \times \frac{L \times N}{60}$

$$= 2 \times \frac{\pi}{4} D^2 \times \frac{LN}{60} = \frac{2ALN}{60}$$

Work done by double acting reciprocating pump = Weight of water delivered $\times$ Total height

$$= \rho g \times \text{Discharge per second} \times \text{Total Height}$$

$$= \rho g \times \frac{2ALN}{60} \times (h_s + h_d)$$

$$\boxed{Q = 2\rho g \times \frac{ALN}{60} \times (h_s + h_d)}$$

$\therefore$ Power required to drive the double-acting pump in kW $(P) = \frac{\text{Work done per second}}{1000}$

$$= \frac{2\rho g \times \frac{ALN}{60} \times (h_s + h_d)}{1000}$$

$$\boxed{P = \frac{2\rho g \times ALN \times (h_s + h_d)}{60{,}000}}$$

A Single-acting reciprocating pump, running at 50 rpm, delivers 0.01 m^3/s of water. The diameter of the piston is 200mm and stroke length 400mm. Determine:

(i) The theoretical discharge of the pump

(ii) Co-efficient of discharge

(iii) Slip and the percentage slip of the pump

Given:

Speed of the pump $N = 50 \text{ rpm}$

Actual discharge $Q_{act} = 0.01 \, m^3/s$

Dia. of piston $D = 200 \, mm = 0.2 \, m$

$\therefore$ Area $A = \frac{\pi}{4}(0.2)^2 = 0.031416 \, m^3$

Stroke $L = 400 \, mm = 0.4 \, m$

(i) Theoretical discharge for single-acting reciprocating Pump

$$Q_{th} = \frac{ALN}{60} = \frac{0.031416 \times 0.4 \times 50}{60}$$

$$= 0.01047 \, m^3/s$$

(ii) Co-efficient of discharge $Cd = \frac{Q_{act}}{Q_{th}} = \frac{0.01}{0.01047} = 0.955$

(iii) $\text{Slip} = Q_{th} - Q_{act} = 0.01047 - 0.01 = 0.00047 \, m^3/s$

$$\% \text{ Slip} = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100 = \frac{(0.01047 - 0.01)}{(0.01047)} \times 100 = \frac{0.00047}{0.01047} \times 100 = 4.489\%$$

· A double acting reciprocating pump, running at 40 rpm, is discharging 1.0 m^3 of water per min. The pump has a stroke of 400 mm. The diameter of the piston is 200 mm. The delivery and suction head are 20 m and 5 m respectively. Find the slip of the pump and power required to drive the pump.

Given : $N = 40$ rpm

Actual discharge, $Q_{act} = 1.0\, m^3/min = \frac{1}{60}\, m^3/s = 0.01666\, m^3/s$

Stroke $L = 400mm = 0.4m$

Diameter of piston (D) = 200 mm = 0.2m

$\therefore$ Area $(A) = \frac{\pi}{4} D^2 = \frac{\pi}{4}(0.2)^2 = 0.031416\, m^2$

Suction head, $h_s = 5m$

Delivery head, $h_d = 20m$

Theoretical discharge for double-acting pump is given by $(Q_{th}) = \frac{2ALN}{60}$

$$= \frac{2 \times 0.031416 \times 0.4 \times 40}{60} = 0.167\, m^3/s$$

Power required to drive the double-acting pump $(P) = \frac{2 \times \rho g \times ALN(h_s + h_d)}{60000}$

$$= \frac{2 \times 1000 \times 9.81 \times 0.031416 \times 0.4 \times 40 \times (5+20)}{60000}$$

$$= 4.109\ kW$$